

**Applied Statistics Comprehensive Examination**

1. Suppose that a random variable  $X$  has the probability density function

$$f(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

- (a) Determine the value of  $c > 0$  for which  $P(X > c) = 0.05$ .  
(b) Find the probability density function of  $Y = X^2$ .
2. An experiment to study the relationship between  $X =$  time spent exercising (min) and  $Y =$  amount of oxygen consumed during the exercise period resulted in the following summary statistics:

$$n = 20 \quad \sum x = 50 \quad \sum y = 16,705$$

$$\sum x^2 = 150 \quad \sum y^2 = 14,194,231 \quad \sum xy = 44,194$$

A scatterplot of the data shows a linear trend.

- (a) Find the equation of the least squares line.  
(b) Test the significance of the slope.
3. The weekly amount of time (in hours) that an industrial machine is down for repairs is a random variable  $X$  with moment-generating function  $M(t) = (1 - 2t)^{-3}$  for  $t < 1/2$ . Suppose that the loss to the industrial operation as a result of the downtime is  $L = 25X + 3X^2$  dollars. Find the expected loss.