Name: ______________________________ Spring, 2020

Applied Statistics Comprehensive Examination

- Calculators are permitted on this examination.
- When you compute a confidence interval, always give an interpretation of the interval in the context of the problem.
- When you perform a hypothesis test, always write down the null and alternative hypotheses, and write the conclusion in the context of the problem.
- There are 200 points on this examination.
- You must give complete explanations to receive full credit.
- Please put your answers and explanations on the separate sheets provided. Please use only the front side of these sheets.

1. (15 points) Suppose that a television critic hypothesizes that the variation in run time per episode of popular cable television shows has been increasing in recent years. She decides to obtain two random samples of 30 episodes from all episodes shown among the 10 most popular "hour-long" cable television shows in 2009 and 2019. The following means and standard deviations are obtained for run times:

Year	n	$\overline{y}$	s
2009	30	51.3	2.1
2019	30	54.2	3.9

   (a) (10 points) Conduct a 0.05 level test to determine if the variance in run times is higher in 2019 than in 2009.

   (b) (5 points) What assumptions are required for this test? Do you feel that these assumptions are reasonable here? Explain briefly.

2. (30 points) Wearing a mask helps to control seasonal influenza virus transmission, but each mask is only effective for a certain period of time. Professor X conducted a two-factor balanced experiment using a completely randomized design to study how the duration of time that a specific mask remains effective changes depending on temperature and humidity levels. She studied two levels of temperature: cold (40 degrees F) and warm (80 degrees F). She also studied two levels of humidity: low (60%) and high (85%). Two masks were tested for each combination of temperature and humidity. The measured times (in hours) are given below. Professor X fit an effects model with interaction.

	Humidity	
Temperature	low	high
cold	10, 16	5, 7
warm	6, 11	3, 5

(a) (10 points) Write down the mathematical model for an effects model with interaction, listing all assumptions and explaining all terms.

(b) (10 points) Create interaction plots and comment on what you see.

(c) (10 points) Using level 0.05, test for interaction.

3. (15 points) For the 2019-2020 year, Villanova published the following enrollment data classifying degree-seeking undergraduate students based on race/ethnicity and class year (rounded to the nearest multiple of 10):

	Race/Ethnicity				
Class Year	White	Hispanic	Asian	Black	Other
First-year	1200	180	120	80	120
Sophomores or higher	3780	380	260	270	390

Conduct a test at the 0.05 level to determine if the distribution of students by race/ethnicity is the same for first-year students and those who are sophomores or higher.

4. (25 points) The Australian fifty-cent coin has the face of Queen Elizabeth II on one side (heads) and the Australian coat of arms on the other side (tails). Suppose that one of these Australian fifty-cent coins was tossed 200 times, producing 84 tails.

   (a) (10 points) Construct a 99% confidence interval for the proportion of times that the Australian fifty-cent coin lands tails up. List any necessary assumptions.

   (b) (10 points) Suppose that an 80% confidence interval for the proportion of times that the Australian fifty-cent coin lands tails up is approximately (38%, 46%). Based on this interval, answer each of the following as "True" or "False".

      i. In this experiment, there is an 80% probability that this coin landed tails up between 38% and 46% of the tosses.

      ii. We can conclude at the 0.2 significance level that this coin does not land heads up 50% of the time.

      iii. The margin of error of this confidence interval is approximately 4%.

      iv. If the Australian fifty-cent coin is tossed another 100 times, we are 80% sure that between 38 and 46 of the tosses will be tails.

      v. Doubling the sample size to 400 would reduce the standard error.

   (c) (5 points) Suppose that the p-value for a test of $H_0 : \pi = 0.5$ vs. $H_0 : \pi \neq 0.5$ is 0.024, indicating that we can conclude that the coin is not fair at the 0.05 level. Carefully interpret the meaning of the p-value, 0.024.

5. (30 points) An ice cream shop owner ran an experiment to study how the melting time for a bowl of ice cream varies depending on the flavor of ice cream and the type of bowl that is used. The owner selected four flavors at random from the many flavors offered by the shop, and he used each of the three types of bowls that the shop owns. For each combination of a flavor and a bowl type, he filled two bowls and recorded the melting time in seconds.

Source	df	SS	MS	Expected MS
Flavor		600		$\sigma^2 + 2\sigma^2_{FB} + 6\sigma^2_F$
Bowl type		200		$\sigma^2 + 2\sigma^2_{FB} + Q(B)$
Interaction		120		$\sigma^2 + 2\sigma^2_{FB}$
Error		60		σ^2

(a) (10 points) Write down an appropriate mathematical model that allows for interaction, listing all assumptions and explaining all terms.

(b) (5 points) Complete the partial ANOVA table given above. Note that $Q(B)$ is a quadratic form in the bowl effects and that σ^2, σ^2_F, and σ^2_{FB} are variance components for error, flavor, and the interaction, respectively.

(c) (15 points) Make appropriate inferences. Specifically, test the significance of all fixed effects using level 0.05, and estimate all variance components.

6. (30 points) A Karnofsky Score ranges from 0 to 100 and is meant to rate a patient's ability to carry out daily activities (with 100 indicating a high ability). In a particular study on 100 lung cancer patients, suppose that each patient and their doctor were asked to assign a Karnofsky score for the patient. The patients assigned themselves an average Karnofsky score of 83 with a standard deviation of 8 while doctors assigned the patients an average Karnofsky score of 84 with a standard deviation of 7. The differences in the 100 scores (patient minus doctor) have a mean of -1 and a standard deviation of 5.

   (a) (10 points) Can we conclude at the 0.05 level that lung cancer patients assign themselves lower Karnofsky scores than their doctors on average? Conduct the appropriate hypothesis test.

   (b) (5 points) What assumptions are needed for the test in part (a)? Do you feel that these assumptions are reasonable here? Explain briefly.

   (c) (5 points) What sample size would be needed if we wanted 80% power for the test in part (a) if the true difference in mean Karnofsky scores (patient minus doctor) is -0.5 and the true standard deviation of the differences is 6.

   (d) (10 points) Suppose that the American Medical Association considers Karnofsky scores for a population of patients unreliable if for a sample of $n \geq 100$ the average sample difference in Karnofsky scores between patient and doctor is at least 2 in absolute value. What is the chance that Karnofsky scores are found to be unreliable based on a sample of 100 lung cancer patients if the true average difference in scores is 1 and the true standard deviation of the differences is 6?

7. (55 points) The World Happiness Report is conducted each year. It compares the level of happiness of people from all the countries of the world, along with other variables. The following analyses were done using a subset of the data in a given year where the case is a country (there is one observation per country). Specifically, they considered the following variables in a linear regression equation to estimate happiness.

Variable	Description
happy	Average happiness score among all respondents in that country. Each person was told "Imagine a ladder, with steps numbered from 0 at the bottom to 10 at the top. The top of the ladder represents the best possible life for you and the bottom of the ladder represents the worst possible life for you. On which step of the ladder would you say you personally feel you stand at this time?"
logGDP	Natural log of gross domestic product per capita (in 2011 US$)
socsup	Proportion of respondents in that country who responded "yes" to the question "If you were in trouble, do you have relatives or friends you can count on to help you whenever you need them, or not?"
sshigh	"TRUE" if socsup for the country was above the median value of socsup "FALSE" if socsup for the country was at or below the median value of socsup

Using the output on pages 9-10, answer the following questions:

(a) (5 points) How many countries were included in Model 1? Show your work.

(b) (5 points) Why is the slope for *logGDP* for Model 1 different than the slope for *logGDP* for Model 2?

(c) Regarding the values of R^2:

  i. (5 points) Identify and interpret the value of R^2 in Model 1.

  ii. (5 points) Using information from Model 1, what are the possible value of the R^2 value for Model 2? Explain your answer.

(d) (15 points) For Model 2, conduct the appropriate hypothesis test for *sshighTRUE*. Explain why this test is of interest.

(e) (15 points) There are two graphs for Model 2. What assumptions can be explored with these graphs? For each assumption, comment on whether it is satisfied based on these graphs.

(f) (5 points) In Model 3, the line *logGDP::sshighTRUE* is the interaction effect between *logGDP* and *sshighTRUE*. Interpret its estimate of 0.42030.

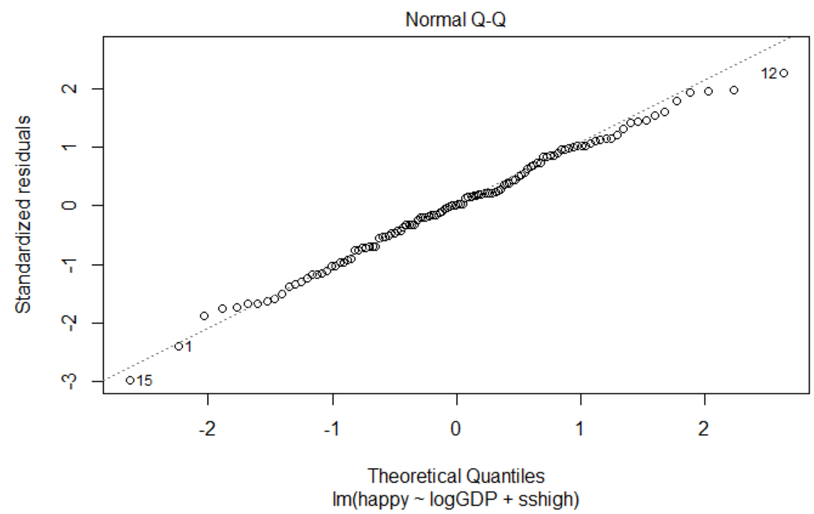
Happiness - Model 1

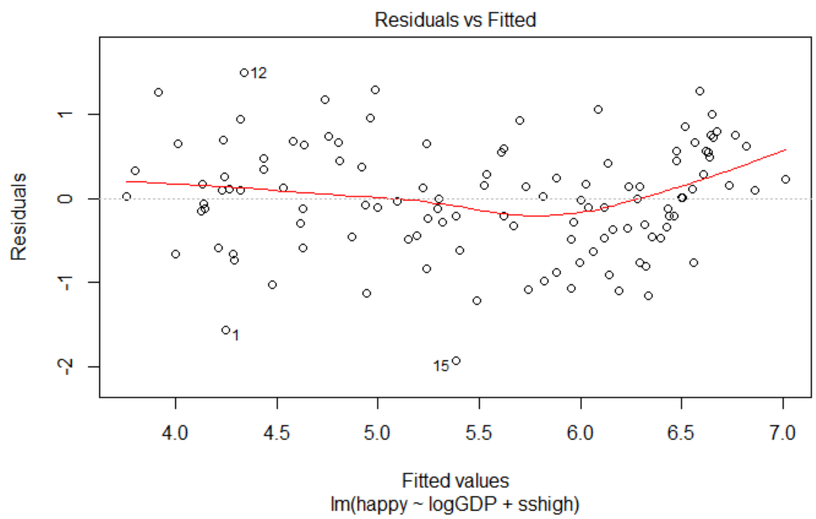
```
Coefficients:
            Estimate Std. Error
(Intercept) -1.15776    0.50853
logGDP       0.72273    0.05458

Residual standard error: 0.704 on 116 degrees of freedom
Multiple R-squared:  0.6019,    Adjusted R-squared:  0.5984
F-statistic: 175.4 on 1 and 116 DF,  p-value: < 2.2e-16
```

Happiness - Model 2

```
Coefficients:
            Estimate Std. Error
(Intercept)   0.3605     0.5937
logGDP        0.5189     0.0699
sshighTRUE    0.7074     0.1661
```





Happiness - Model 3

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)        1.56247    0.70520   2.216  0.02870 *  
logGDP             0.37580    0.08334   4.509 1.59e-05 ***
sshighTRUE        -3.27449    1.36280  -2.403  0.01789 *  
logGDP:sshighTRUE  0.42030    0.14284   2.942  0.00395 ** 
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```